



THEORY OF ELASTICITY AND PLASTICITY

1. The system of equations
$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + X = 0$$
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + Y = 0$$
 represents:

- a) the static equilibrium equations for an infinitesimal element detached from a body subjected to a plane stress state;
- b) the dynamic equilibrium equations for an infinitesimal element detached from a body subjected to a plane stress state;
- c) the boundary conditions in plane elasticity;
- d) the continuity condition in plane elasticity.

The stress tensor at a point of a deformable loaded is:

2.
$$T_{\sigma} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} = \begin{bmatrix} 20 & 0 & 4 \\ 0 & 10 & 0 \\ 4 & 0 & -12 \end{bmatrix} \text{ N/mm}^2$$

What is the tensor element that represents a principal stress?

- a. 4 N/mm²
- b. 10 N/mm²
- c. -12 N/mm²
- d. 20N/mm²

3. The system of equations
$$p_x = \sigma_x l + \tau_{yx} m$$
$$p_y = \tau_{xy} l + \sigma_y m$$
 represents:

- a) the static equilibrium equations for an infinitesimal element detached from a body subjected to a plane stress state;
- b) the dynamic equilibrium equations for an infinitesimal element detached from a body subjected to a plane stress state;
- c) the boundary conditions in plane elasticity;
- d) the continuity condition in plane elasticity the continuity.

4. The system of equations $\epsilon_x = \frac{\partial u}{\partial x}$, $\epsilon_y = \frac{\partial v}{\partial y}$, $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$, represents:

- a) the constitutive law of the material in plane elasticity;
- b) the geometric equations in plane elasticity;
- c) the constitutive law of the material in three-dimensional elasticity;
- d) the boundary conditions in plane elasticity.

The strain tensor at a point of a homogeneous and isotropic body is:

$$5. \quad T_\varepsilon = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{yx} & \frac{1}{2}\gamma_{zx} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{zy} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix} = 10^{-4} \begin{bmatrix} 8 & 3 & 0 \\ 3 & -4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

A principal direction of deformation at this point coincides to:

- a. direction of axis Ox b. direction of axis Oy c. direction of axis Oz d. bisector of angle $x\hat{O}y$

6. Condition $\Delta(\sigma_x + \sigma_y) = 0$ represents:

- a) a static equilibrium equation in plane elasticity;
b) the continuity condition expressed in terms of stresses, in plane elasticity;
c) the continuity condition expressed in terms of stresses, in three-dimensional elasticity;
d) a boundary condition in plane elasticity.

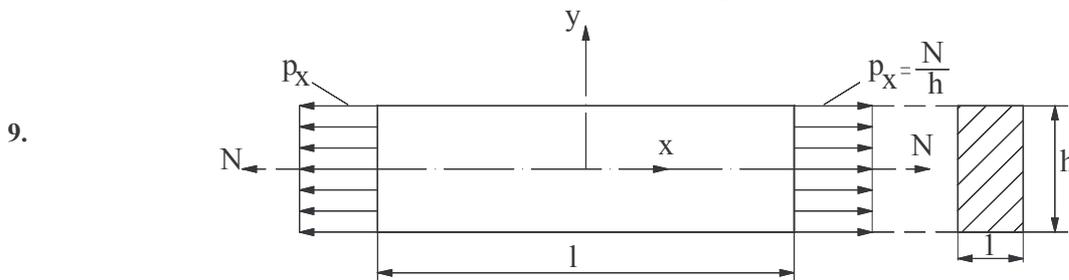
7. The solution of a plane elasticity problem in terms of stresses, by considering a Cartesian coordinate system, consists in solving the differential equation (notation $\nabla^2 \nabla^2 \equiv \Delta \Delta$):

- a. $\nabla^2 \nabla^2 w(x, y) = \frac{p(x, y)}{D}$ b. $\nabla^2 \nabla^2 F(r, \vartheta) = 0$ c. $\frac{d^4 w}{dx^4} = \frac{p(x)}{D}$ d. $\nabla^2 \nabla^2 F(x, y) = 0$

8. The stress function $F(x, y)$ generates the following stresses:

- a) $\sigma_x = \frac{\partial^2 F}{\partial x^2}$; $\sigma_y = \frac{\partial^2 F}{\partial y^2}$; $\tau_{xy} = \frac{\partial^2 F}{\partial x \partial y}$;
b) $\sigma_x = \frac{\partial F}{\partial y}$; $\sigma_y = \frac{\partial F}{\partial x}$; $\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y}$;
c) $\sigma_x = \frac{\partial^2 F}{\partial y^2}$; $\sigma_y = \frac{\partial^2 F}{\partial x^2}$; $\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y}$;
d) $\sigma_x = \frac{\partial^2 F}{\partial x^2} - X \cdot x$; $\sigma_y = \frac{\partial^2 F}{\partial y^2} - Y \cdot y$; $\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y}$;

For the rectangular two-dimensional element shown in the figure, the stress function is:



- a. $F(x) = p_x \frac{x^2}{2}$ b. $F(x, y) = p_x xy$; c. $F(y) = p_x \frac{y^2}{2}$; d. $F(y) = p_x \frac{y^3}{6}$

10. The polynomial corresponding to tension along to orthogonal directions is:

- a. $F(x, y) = \frac{ax^2}{2} + bxy$ b. $F(x, y) = \frac{ax^3}{6} + \frac{cy^3}{6}$ c. $F(x, y) = bxy + \frac{cy^2}{6}$ d. $F(x, y) = \frac{ax^2}{2} + \frac{cy^2}{2}$

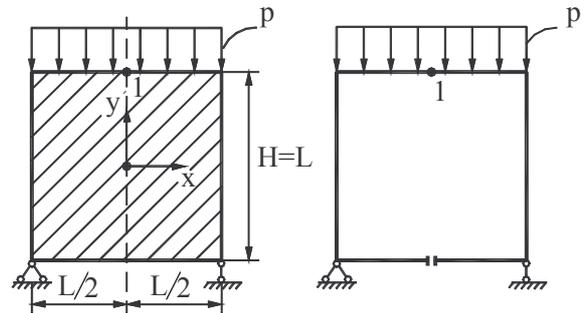
11. The polynomial $F(x, y) = \frac{ay^2}{2} + \frac{dy^3}{6}$ corresponds to:

- a. eccentric tension in y axis direction b. eccentric tension in x axis direction c. combined shear and bending d. tension along two directions

12. The stresses generated by the polynomial $F(x, y) = \frac{ax^2}{2} + bxy + \frac{cy^3}{6}$ have the expressions:

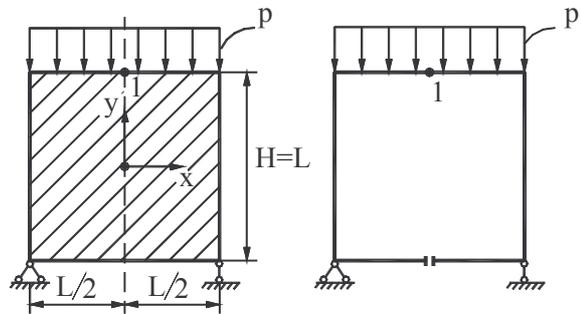
- a. $\sigma_x = cy; \sigma_y = a; \tau_{xy} = -b$ b. $\sigma_x = bx + cy; \sigma_y = a; \tau_{xy} = -b$ c. $\sigma_x = cy; \sigma_y = 0; \tau_{xy} = -by$ d. $\sigma_x = bx + cy; \sigma_y = a; \tau_{xy} = 0$

13. The stress function at point "1" of the two-dimensional element with unit thickness, shown in the figure, is:



- a. $F_1 = -pL$ b. $F_1 = \frac{pL^2}{4}$ c. $F_1 = -\frac{pL^2}{2}$ d. $F_1 = \frac{pL^2}{8}$

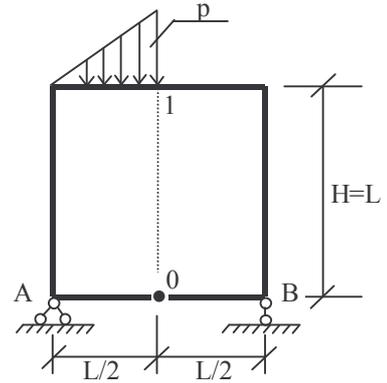
14. The correct values of the stresses σ_y and τ_{xy} at point "1" of the element presented in the figure are:



- a. $\sigma_y = p, \tau_{xy} = 0$ b. $\sigma_y = -p, \tau_{xy} = p$ c. $\sigma_y = -p, \tau_{xy} = 0$ d. $\sigma_y = 0, \tau_{xy} = 0$

15.

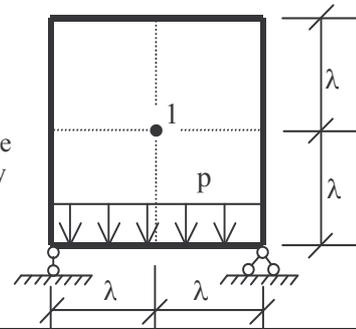
The stress function at point "1" of the two-dimensional element shown in the figure (when the origin is located at point O) is:



- a. $F_1 = \frac{pL^2}{12}$ b. $F_1 = \frac{pL^2}{24}$ c. $F_1 = -\frac{pL^2}{6}$ d. $F_1 = -\frac{pL^2}{24}$

16.

What are the values of the stresses σ_x , σ_y , τ_{xy} at the central point of the deep beam shown in the figure, when their evaluation is performed by using the finite differences method, with the presented grid?



- a) $\sigma_x = \frac{1}{8}p$, $\sigma_y = \frac{1}{3}p$, $\tau_{xy} = 0$
 b) $\sigma_x = -\frac{1}{6}p$, $\sigma_y = \frac{1}{3}p$, $\tau_{xy} = 0$
 c) $\sigma_x = \frac{1}{2}p$, $\sigma_y = \frac{1}{4}p$, $\tau_{xy} = 0$
 d) $\sigma_x = \frac{1}{3}p$, $\sigma_y = \frac{1}{4}p$, $\tau_{xy} = 0$

17.

Mention the value for the ratio L/H for which a rectangular two-dimensional element, loaded in its middle surface plane, is considered to be a deep beam:

- a. $\frac{L}{H} = 10$ b. $\frac{L}{H} < 5$ c. $\frac{L}{H} > 5$ d. $\frac{L}{H} > 10$

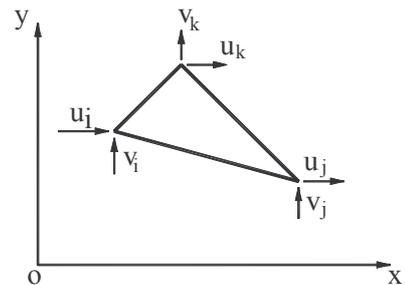
18.

For the two-dimensional triangular finite element shown in the figure, the displacement field can be expressed as:

$$u(x,y) = N_i u_i + N_j u_j + N_k u_k$$

$$v(x,y) = N_i v_i + N_j v_j + N_k v_k$$

where N_i , N_j , N_k are:

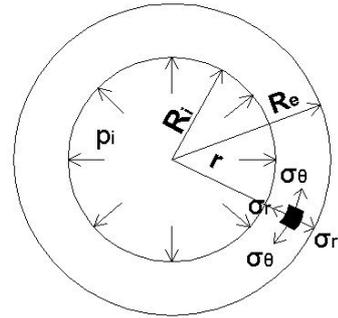


- a. stress functions b. axial forces c. Weighting functions d. shape functions

The stresses produced by the interior pressure p_i in a cylinder with thick walls are:

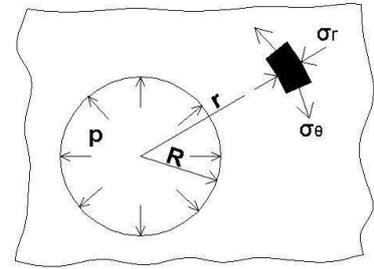
$$\sigma_r = 2A + \frac{B}{r^2}; \quad \sigma_\theta = 2A - \frac{B}{r^2};$$

19. The constants $2A$ and B are obtained by using the following boundary conditions:



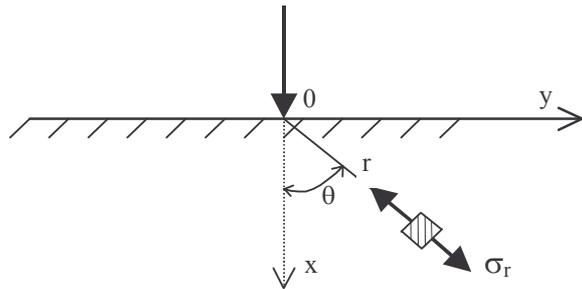
- a. $(\sigma_r)_{r=R_i} = p_i;$
 $(\sigma_r)_{r=R_e} = 0$
- b. $(\sigma_r)_{r=R_i} = -p_i$
 $(\sigma_\theta)_{r=R_e} = p_i$
- c. $(\sigma_\theta)_{r=R_i} = 0$
 $(\sigma_r)_{r=R_e} = 0$
- d. $(\sigma_r)_{r=R_i} = -p_i$
 $(\sigma_r)_{r=R_e} = 0;$

20. The stresses at a point of a infinite plate with a circular hole acted by a constant radial pressure p are:



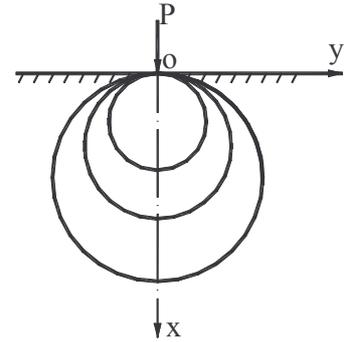
- a. $\sigma_r = -p\left(\frac{R}{r}\right)^2$
 $(\sigma_\theta) = p\left(\frac{r}{R}\right)^2$
- b. $\sigma_r = -p\left(\frac{R}{r}\right)^2$
 $(\sigma_\theta) = 0$
- c. $\sigma_r = p\left(\frac{R}{r}\right)^2$
 $(\sigma_\theta) = 0$
- d. $\sigma_r = -\sigma_\theta = -p\left(\frac{R}{r}\right)^2$

21. At a point of an elastic half-plane, loaded by a force that acts normal to the surface, as presented in the figure, the radial stresses are:



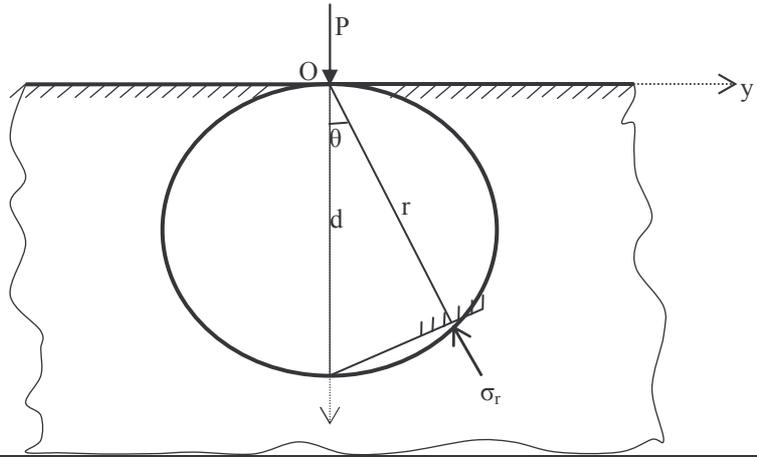
- a. $\sigma_r = \frac{2P \cos \theta}{\pi r}$
- b. $\sigma_r = -\frac{2P \cos \theta}{\pi r}$
- c. $\sigma_r = -\frac{3P \cos \theta}{2\pi r}$
- d. $\sigma_r = -\frac{3P \cos \theta}{2\pi r^2}$

22. For an elastic half-plane, loaded by a force normal to the boundary, as shown in the figure, the circles tangent to the boundary at the origin are called:



- a. isochromatics b. isoclines c. trajectories of first kind d. isobars

23. Knowing $\sigma_r = -\frac{2P}{\pi} \cdot \frac{\cos\theta}{r}$ for a half-plane acted by a force normal to the boundary, mention the values that characterize the isobars:



- a. $-\frac{2P}{\pi r}$ b. $\frac{2P}{\pi r}$ c. $-\frac{2P}{\pi d}$ d. $-\frac{P}{\pi d}$

24. The differential equation of the deformed middle surface in Cartesian coordinates for rectangular plates, acted by normal forces to their middle plane, has the shape:

- a) $\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0;$
 b) $\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p(x, y)}{D};$
 c) $\frac{d^4 w}{dr^4} + \frac{2}{r} \frac{d^3 w}{dr^3} - \frac{1}{r^2} \frac{d^2 w}{dr^2} + \frac{1}{r^3} \frac{dw}{dr} = \frac{p(r)}{D};$
 d) $\frac{d^4 w}{dx^4} = \frac{p(x)}{EI};$

25. The internal forces that occur in rectangular plates, loaded by normal forces to their middle plane, are:

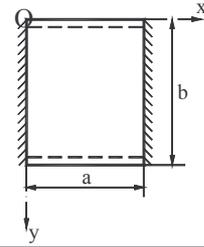
- a) the axial forces N_x, N_y ; the shear forces T_x, T_y ;
 b) the axial forces N_x, N_y ; the bending moments M_x, M_y ;
 c) the bending moments M_x, M_y ; the twisting moment $M_{xy} = M_{yx} = M_t$; the shear forces T_x, T_y ;
 d) the axial forces N_x, N_y ; the twisting moment $M_{xy} = M_{yx}$;

26. Some of the stresses that occur in a plate subjected to bending have maximum absolute value on the upper surface and the lower surface of the plate. What are these stresses?

- a. $\sigma_x, \tau_{xz}, \tau_{yz};$ b. $\sigma_y, \tau_{xz}, \tau_{yz};$ c. $\sigma_x, \sigma_y, \tau_{xy} = \tau_{yx};$ d. $\tau_{xy}, \tau_{xz}, \tau_{yz};$

29.

The boundary conditions for the rectangular plate shown in the figure are:



a.

on sides $x = 0$;
 $x = a \begin{cases} w = 0 \\ \frac{\partial w}{\partial x} = 0 \end{cases}$
 on sides $y = 0; y = b \begin{cases} w = 0 \\ \frac{\partial w}{\partial y} = 0 \end{cases}$

b.

on sides $x = 0$;
 $x = a \begin{cases} w = 0 \\ \frac{\partial w}{\partial x} = 0 \end{cases}$
 on sides $y = 0$;
 $y = b \begin{cases} w = 0 \\ \frac{\partial^2 w}{\partial y^2} = 0 \end{cases}$

c.

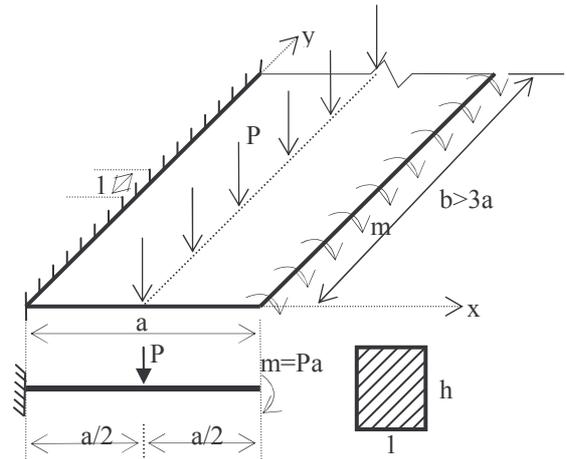
on sides $x = 0$;
 $x = a \begin{cases} w = 0 \\ \frac{\partial^2 w}{\partial x^2} = 0 \end{cases}$
 on sides $y = 0$;
 $y = b \begin{cases} w = 0 \\ \frac{\partial^2 w}{\partial y^2} = 0 \end{cases}$

d.

On sides $x = 0$;
 $x = a \begin{cases} w = 0 \\ \frac{\partial^2 w}{\partial x^2} = 0 \end{cases}$
 on sides $y = 0$;
 $Y = b \begin{cases} w = 0 \\ \frac{\partial w}{\partial y} = 0 \end{cases}$

30.

The extreme normal stresses extreme $\sigma_{x \max / \min}$ for the rectangular plate shown in the figure are:



a.

$\pm \frac{P}{h^2}$

b.

$\pm \frac{3Pa}{2h^2}$

c.

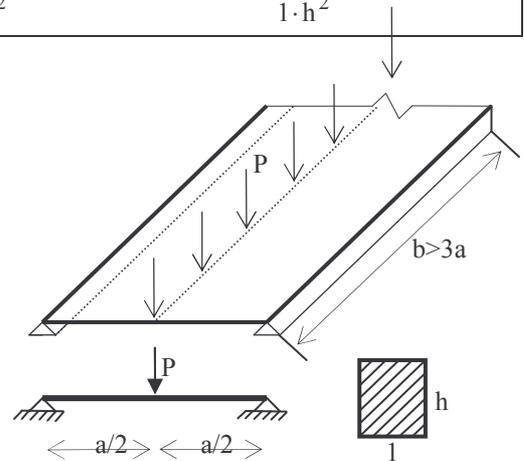
$\pm \frac{9Pa}{1 \cdot h^2}$

d.

$\pm \frac{6Pa}{1 \cdot h^2}$

31.

What is the thickness h of the plate presented in the figure and made of a material characterized by the limit normal stress σ_0 ?



a.

$\sqrt{\frac{Pa}{\sigma_0}}$

b.

$\sqrt{\frac{3Pa}{2\sigma_0}}$

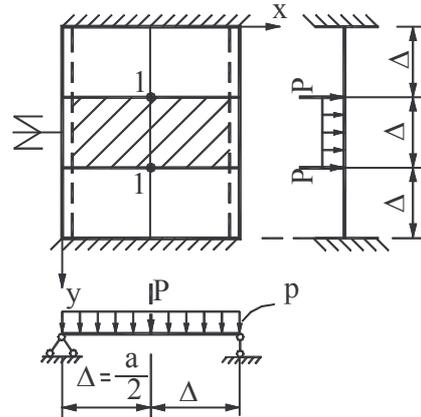
c.

$\sqrt{\frac{6Pa}{\sigma_0}}$

d.

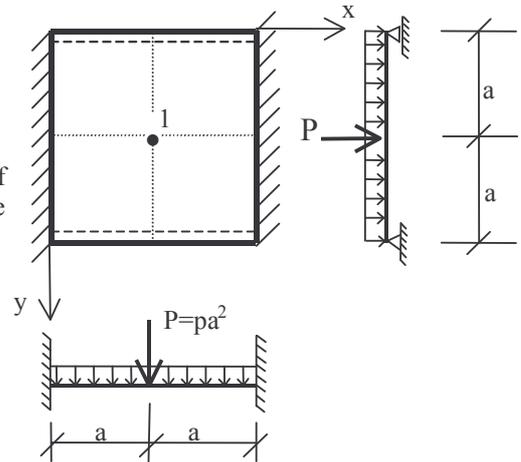
$\sqrt{\frac{Pa}{2\sigma_0}}$

32. For the plate loaded as shown in the figure, indicate the correct value of the free term p_1 , resulted from the transcription into finite differences of equation $\nabla^2 \nabla^2 w(x, y) = \frac{p(x, y)}{D}$, at point 1.



- a. $p_1 = p + \frac{P}{\Delta^2}$; b. $p_1 = \frac{p}{2} + P$; c. $p_1 = \frac{p}{2} + \frac{P}{\Delta^2}$; d. $p_1 = p + P$;

33. The deflection and bending moments at the central point of the plate presented in the figure, determined by using the finite differences method for the indicated grid, are:



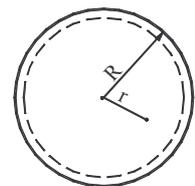
- a. $w_1 = \frac{pa^4}{10D}$, $M_{x1} = M_{y1} = \frac{1+v}{5} pa^2$ b. $w_1 = \frac{pa^4}{5D}$, $M_{x1} = \frac{1+v}{5} pa^2$, $M_{y1} = \frac{1+v}{10} pa^2$ c. $w_1 = \frac{pa^4}{4D}$, $M_{x1} = \frac{pa^2}{5}$, $M_{y1} = \frac{pa^2}{10}$ d. $w_1 = \frac{pa^4}{20D}$, $M_{x1} = \frac{pa^2}{10}$, $M_{y1} = \frac{pa^2}{5}$

34. In the axial-symmetric circular and annular plates, the following internal forces occur:

- a. $M_r, M_{r\theta} = M_{\theta r}, T_r$; b. $M_\theta, M_{r\theta} = M_{\theta r}, T_\theta$; c. $M_r, M_\theta, M_{r\theta} = M_{\theta r}$ d. M_r, M_θ, T_r ;

35. For a circular solid plate, the solution of equation $\nabla^2 \nabla^2 w(r) = \frac{p(r)}{D}$, has the form:

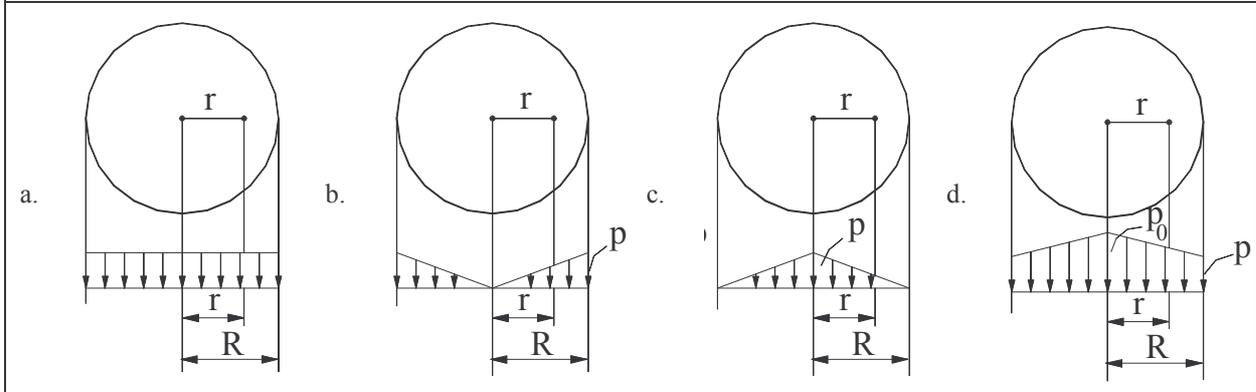
- $w = A_1 + B_1 r^2 + w_p$. Mention the boundary conditions corresponding to a plate simply supported along the whole contour, needed to determine the integration constants A_1 and B_1 :
For $r = R$



- a. $w=0$; $\frac{dw}{dr}=0$ b. $w=0$; $\frac{d^2w}{dr^2} + \frac{v}{r} \frac{dw}{dr} = 0$ c. $w=0$; $\frac{d^2w}{dr^2} = 0$ d. $w=0$; $\frac{d^2w}{d\theta^2} = 0$

Indicate the loading case for the circular plate, knowing that the corresponding particular solution is

36. $w_p = \frac{pr^4}{64D}$:

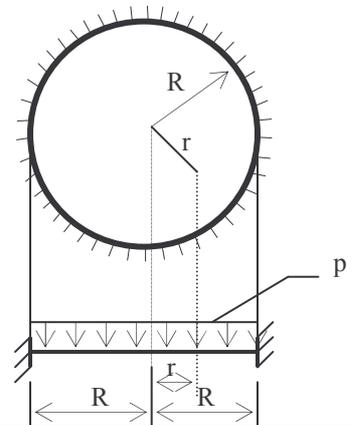


The deflection expression for the circular plate presented in the figure is

37. $w(r) = A + Br^2 + \frac{pr^4}{64D}$.

The maximum deflection is:

37.



- a. $\frac{pR^4}{24D}$ b. $\frac{pR^2}{32D}$ c. $\frac{pR^4}{16D}$ d. $\frac{pR^4}{64D}$

38. What is the condition that does not correspond to membrane state for shells?:

- a) the shell thickness, constant or slow variable, is small;
- b) the shell surface is continuous (without holes or stiffenings etc.);
- c) the shell is continuously supported in the tangent plane to the middle surface;
- d) the loads (forces or moments) are concentrated and can have any sense.

For axial-symmetric rotation shells, in the membrane theory, the internal force in the meridian direction,

39. N_φ , at a section defined by the angle φ is expressed by using the relation $N_\varphi = -\frac{R \Delta\varphi}{2\pi r \sin \varphi}$, where $R \Delta\varphi$ is:

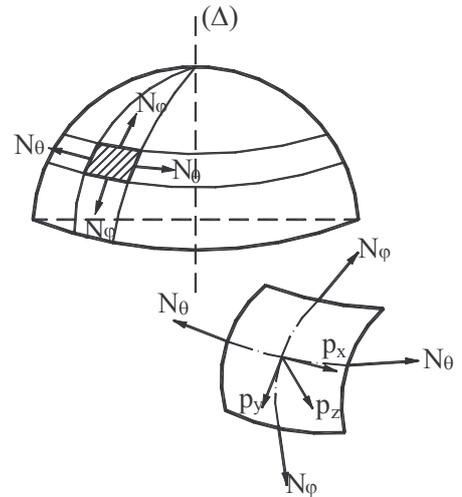
- a) the radius of curvature;
- b) the reaction along the boundary;
- c) the resultant of the afferent gravitational loads;
- d) the resultant of reactions along the boundary.

40. Over the thickness of the shells that work in membrane state, the stresses are:

- a. zero b. uniform distributed c. linear distributed d. parabolic distributed

41.

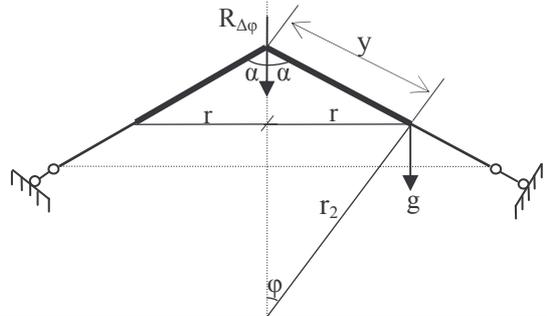
The circumferential internal force N_θ , in axial-symmetric rotation shells, according to membrane state theory, is obtained from an algebraic equilibrium equation that has the form: (r_1, r_2 – the principal radii of curvature at a point of the surface)



- a. $\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} + p_x = 0$ b. $\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} + p_z = 0$ c. $\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} + p_y = 0$ d. $\frac{N_\phi}{r_2} + \frac{N_\theta}{r_1} + p_z = 0$

42.

The resultant $R_{\Delta\phi} = R_{\Delta y}$ of the load given by the own weight (g – the weight per unit surface), at a current section of the conical dome shown in the figure is:



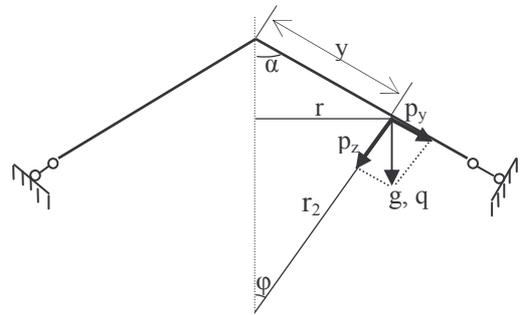
- a. $g \cdot \pi r^2$ b. $g \cdot \pi r l$ c. $g \cdot \pi r y$ d. $g \cdot 2\pi r y$

43. The components of the load produced by snow on axial-symmetric rotation shells are:

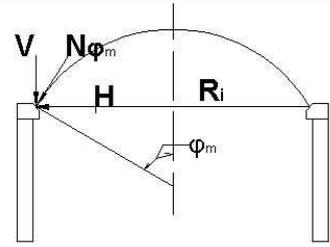
- a. $p_x = 0,$
 $p_y = g \cdot \sin \phi,$
 $p_z = g \cdot \cos \phi$ b. $p_x = 0,$
 $p_y = 0,$
 $p_z = \gamma \cdot H_\phi$ c. $p_x = 0,$
 $p_y = q \cdot \cos \phi \cdot \sin \phi,$
 $p_z = q \cdot \cos^2 \phi$ d. $p_x = 0,$
 $p_y = 0,$
 $p_z = p \cdot \sin \phi \cdot \cos \theta.$

44.

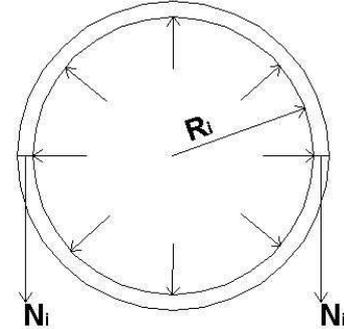
The membrane internal force N_θ , at a current section of the conical dome presented in the figure, is determined from an algebraic equation that has the form:



- a. $\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} + p_y = 0$ b. $\frac{N_\phi}{r_2} + \frac{N_\theta}{r_1} + p_z = 0$ c. $\frac{N_\theta}{r_2} + p_y = 0$ d. $\frac{N_\theta}{r_2} + p_z = 0$



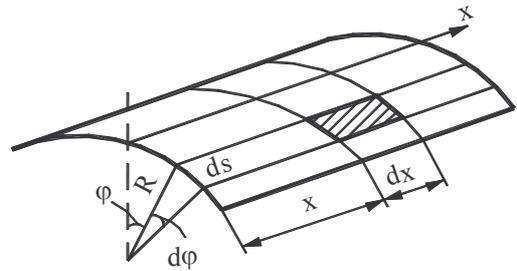
45. The internal force in the supporting ring of the dome shown in the figure is:



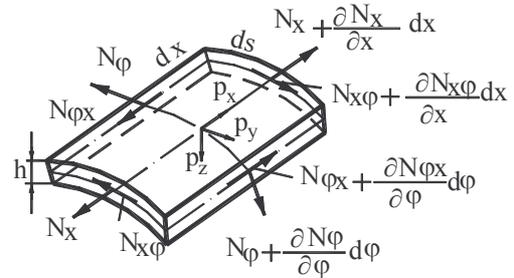
- a. $V \cdot R_i$ b. $N_{\phi m} \cdot R_i$ c. $H \cdot R_i$ d. $-H \cdot R_i$

The equilibrium equations for open cylindrical shells that work in membrane state are:

46.
$$\begin{cases} \frac{\partial N_x}{\partial x} + \frac{1}{R} \frac{\partial N_{\phi x}}{\partial \phi} + p_x = 0; \\ \frac{\partial N_{x\phi}}{\partial x} + \frac{1}{R} \frac{\partial N_\phi}{\partial \phi} + p_y = 0; \\ \frac{N_\phi}{R} + p_z = 0. \end{cases}$$

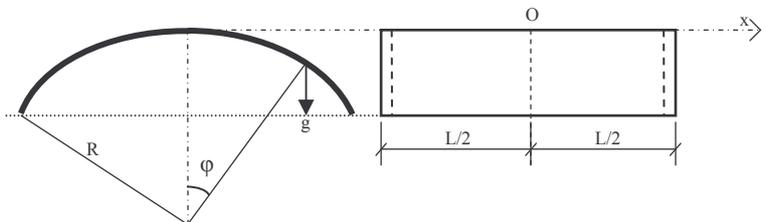


The internal forces are obtained from this system in the following order:



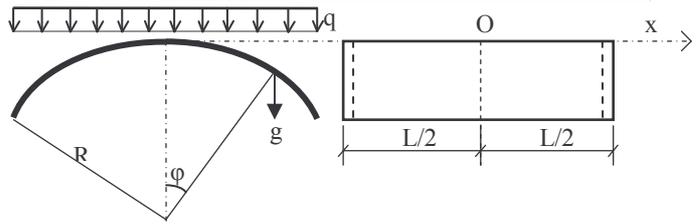
- a. $N_x, N_\phi, N_{x\phi}$ b. $N_x, N_{x\phi}, N_\phi$ c. $N_\phi, N_{x\phi}, N_x$ d. $N_\phi, N_x, N_{x\phi}$

47. In the cylindrical shell having one span and one bay, supported on a pediment, the internal force N_ϕ produced by the own weight has the expression:



- a. $-2gx \cdot \sin \phi$ b. $2gR \cdot \cos \phi$ c. $-\frac{g}{R} \left(\frac{L^2}{4} - x^2 \right) \cdot \cos \phi$ d. $-gR \cdot \cos \phi$

48. The internal force N_φ , produced by the weight of snow in a cylindrical roof with one span and one bay, supported on a pediment, is determined by using the relation:



- a. $-qR \cdot \cos^2 \varphi$ b. $qR \cdot \cos \varphi$ c. $-\frac{3}{2}qx \cdot \sin 2\varphi$ d. $-\frac{3}{2} \frac{q}{R} \left(\frac{L^2}{4} - x^2 \right) \cdot \cos 2\varphi$

49. The yielding criterion at a point where the stresses σ and τ are known, can be expressed as:

- a. $\sigma_1 = \sigma_c$ b. $\frac{1}{2} \sqrt{\sigma^2 + \tau^2} = \sigma_c$ c. $\sqrt{\sigma^2 + 4\tau^2} = \sigma_c$ d. $\sqrt{\sigma^2 + \tau^2} = \sigma_c$

50. The yielding criterion *Von Mises* for the stress state at a point, expressed by the stresses σ and τ , has the form:

- a. $\sqrt{\sigma^2 + 2,6\tau^2} = \sigma_c$ b. $\sqrt{\sigma^2 + 3\tau^2} = \sigma_c$ c. $\sqrt{\sigma^2 + \tau^2} = \sigma_c$ d. $\frac{1}{2} \sqrt{\sigma^2 + 4\tau^2} = \sigma_c$